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The boundary value problems for thin coats

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Abstract

We formulate and solve three boundary value problems for the coats of finite and infinite domains in the plane.

Keywords: Displacements; Analytic functions; Hilbert problem; Boundary conditions

1. Introduction

The linear dependence on the coordinate $\tilde{z}$ of the displacements vector in a thin coat of a finite domain in the horizontal XOY plane was considered in the paper (Shirokova, 2003). There the boundary value problem with the solvability condition was formulated and solved. Here we formulate new boundary value problems with boundary conditions which differ from those in Shirokova (2003) for the coats of finite and infinite domains in the plane. These problems are applicable to the "soft" elastic layers of a "sandwich" structure.

2. Analysis

Let D be a one-connected domain in the plane XOY of $XY\tilde{Z}$ space. We use the same assumption for the displacements in the coat of D as in Shirokova (2003): if $\vec{u} = (u_1, u_2, u_3)$ is the vector of displacements then

$$u_1 = \hat{u}_1(x,y) + \tilde{z}\tilde{u}_1(x,y), u_2 = \hat{u}_2(x,y) + \tilde{z}\tilde{u}_2(x,y), u_3 = \tilde{z}\tilde{u}_3(x,y),$$

the derivatives of $\hat{u}_j$, $\tilde{u}_j$ being continuous in the closure of D. Now if we find the components of the stress tensor and put them in the equilibrium equations we obtain these equations in the form $A_k(x,y) + \tilde{z}B_k(x,y) = 0, k = 1, 2, 3$, for all $(x,y) \in D$ and $\tilde{z} \in [0, \tilde{z}(x,y)]$, $\tilde{z} = \tilde{z}(x,y), (x,y) \in D$, being the equation of the upper surface of the coat. So we suppose that $A_k(x,y) = 0$, $B_k(x,y) = 0$, $k = 1, 2, 3$. The last equations allow the analytic functions introduction and lead in Shirokova (2003) to the following relations:

$$\hat{u}_1(x,y) + \imath\hat{u}_2(x,y) = -\frac{1}{2\mu}(z\overline{\phi'(z)} + \overline{\psi(z)}) + q(z), \tag{1}$$

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